# Determination and Collection of Non - Cyclic Group Conjectures 

RODELIO M. GARIN<br>Pangasinan State University, Asingan, Pangasinan<br>rodgarin36@gmail.com


#### Abstract

This study aims to determine and collect samples of the non-cyclic group. This study was carried out by a process of investigations. The investigation focuses on the Set together with the Operation, then determine if the pair satisfies non-cyclic group properties. Concerning the result, the following conjectures were derived: (1) the largest number of generators for the set of non - zero residue classes modulo $n$ relatively prime to $n$ under operation multiplication is two. (2) The non - zero residue classes modulo $n=2^{r}$ where $r \geq 3$ relatively prime to $n$ with operation multiplication are non - cyclic groups.(3) If the order of $G_{1}$ is one and $G_{2}$ is a cyclic group, then $\prod_{i=1}^{2} G_{i}$ is cyclic group under the operation direct product of the groups $G_{i}$. (4) If the order of $G_{1}$ is one and $G_{2}$ is a non-cyclic group, then $\prod_{i=1}^{2} G_{i}$ is non-cyclic group under the operation direct product of the groups $G_{i}$. (5) If at least two of the $G_{i}$ has an order greater than 1. Then, $\prod_{i=1}^{n} G_{i}$ is non-cyclic group under the operation direct product of the groups $G_{i}$ whenever $G_{i}$ are cyclic or non cyclic group. Those conjectures formulated were verified correct. Therefore, the study recommends that the teachers may use those conjectures as an example in Abstract Algebra Subject specifically cyclic and non-cyclic group.


Keywords - Investigate; Set and Operation; Non-Cyclic Properties

## I. INTRODUCTION

The most important structures of Abstract Algebra are groups, rings and fields. Groups are classified into two the Cyclic and Non-cyclic Groups [1]. This study of algebraic systems is designed for the investigation of the given Set and Operation if they form a non-cyclic group. Non-cyclic is one of the major concepts for a course Abstract Algebra which have least classification sample problems. The basic components of algebraic system that were used in this study were Set and Operations. The investigation focuses on these two components, then determine the pairs which satisfy the non- cyclic group properties.

A cyclic group as "a set $S$ with the given operation is a cyclic group if there exists an element $\alpha \in S$ generates the set $S$, and will be denoted by $\langle\alpha\rangle$ " ${ }^{[2]}$. If there is no such element $\alpha$ of S that generates set $S$ then $S$ with the given operation is called non - cyclic group.

The results of this study will be serving as an example, or exercises to enhance students' learning about algebraic system specifically cyclic and non cyclic groups. The teachers teaching abstract algebra and other related subjects may have an additional classification of example regarding this topic.

The development of this study was supplemented by learning theories and some concepts
and theorems on the algebraic systems. The following theories, concepts, generalizations, and ideas serve as a guide for the conduct of this study.

Constructivism is the philosophy that the learner can construct their own understanding based on their experiences. Each of us are learners and it is possible that we can generate models using our own experience to create new ideas which facilitate learning. It is suggested that learners should explore their previous knowledge by constructing, using investigation or experimentation to create more knowledge [3].

Learners are encouraged to learn if there isa new model that they will study which they can acquire new knowledge. The center of intrinsic motivation is curiosity that means the desire to know much more [4].

One of the concepts in Abstract Algebra subject which inspired the researchers to pursue this study is the cyclic subgroups. A group $G$ is called cyclic if, there exist $\alpha \in G$, such that every $x \in G$ is of the form $\alpha^{m}$, where $m \in Z$. The element $\alpha$ is called the generator [2].

Another algebraic system theorem which also used in the investigation is the direct product of the groups. The theorem is stated as "If $G_{1}, G_{2}, \ldots, G_{n}$ be groups. For $\left(a_{1}, a_{2}, \ldots, a_{n}\right)$ and $\left(b_{1}, b_{2}, \ldots, b_{n}\right)$ in $\prod_{i=1}^{n} G_{i}$, define $\left(a_{1}, a_{2}, \ldots, a_{n}\right)\left(b_{1}, b_{2}, \ldots, b_{n}\right)$ to be the element $\left(a_{1} b_{1}, a_{2} b_{2}, \ldots, a_{n} b_{n}\right)$. Then $\prod_{i=1}^{n} G_{i}$ is a
group, the direct product of the groups $G_{i}$, under this binary operation [5]. This study also focuses its investigation regarding direct product groups. Then, observe which of the sets with operation multiplication satisfy the property of non - cyclic groups.

## II. METHODS

## Research Design

This study was carried out by mathematical investigation. Mathematical investigation is an approach to increase emphasis of problem solving processes. It is considered as a tool for mathematical processes in the learner and an exercise of intellect to unearth mathematical structure from a simple starting point [6].

## Procedure

This study used five stages of mathematics of Investigation (see figure 1).

## i. Identifying Set and Operation

The first stage is on Identifying set and operation. During this stage, the question what set and operation should be considered that satisfy the non-cyclic grouped were outlined. At this stage also, the researcher list and collect sets and operations that were used in the investigation.

## ii. Exploring Systematically

In this stage the collected sets and operations will be paired, each pair of the collected sets and operations will be observed, if the operation and the set satisfy group property but doesn't have a generator then it is a non-cyclic group.

## iii. Initial Conjecture

The data obtained were presented using table. Patterns in the data were considered then make a generalization. The generalization which obtained inductively was considered as the initial conjecture. Each pair of operation and set considered in this study were used to formulate conjectures.

## iv. Testing Conjectures

This stage the initial conjecture formulated would be checked its consistency by giving an additional illustration beyond the previous illustrations. The illustration may support the conjecture or provide a counter - example indicating the need to revise or reject the initial conjecture.

Figure 1. Circular Flow Chart of the Investigation
v. Final Conjecture


The generalization which supported by the illustrations and tested its consistency was the final conjectures. The final conjectures found in this study were not proved, but checked consistently.
Figure 1 shows the flow chart of the investigation used in this study.

## III. RESULTS AND DISCUSSION

This section of the study, presented some classifications of non-cylic group and the pairs of set and operation that were investigated. The order of each element of the set was determined and listed to make generalizations. The conjectures were also listed after the investigation from each pair of set and operation.

Viergrouppe. One of the famous examples of non - cyclic groups are Viergrouppe. The Viergrouppe is a set $V=\{e, a, b, c\}$ with $e \epsilon V$ as the identity element. Figure 2 shows the multiplication table of Viergrouppe.

|  | $e$ | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: | :---: |
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $e$ | $a$ |
| $c$ | $c$ | $b$ | $a$ | $e$ |

Figure 2. Multiplication Table of Viergrouppe
The order of the elements of V we have;

$$
\begin{aligned}
& \langle e\rangle=\{e\} \rightarrow|\langle e\rangle|=1 \\
& \langle a\rangle=\{e, a\} \rightarrow|\langle a\rangle|=2 \\
& \langle b\rangle=\{e, b\} \rightarrow|\langle b\rangle|=2 \\
& \langle c\rangle=\{e, c\} \rightarrow|\langle c\rangle|=2
\end{aligned}
$$

The order of each element of $V$ is less than 4. Therefore, Viergrouppe is called non - cyclic group.

Symmetric Permutation. Another familiar example of non - cyclic group is the symmetric permutation. Let consider the set whose elements is Permutation of the elements of the given set denoted by $S_{n}$. Let $A=\{1,2,3\}$, if $S_{3}$ be the collection of the permutations of the elements of $A$. Then, the order of $S_{3}$ we have,

$$
S_{3}=3!=6
$$

Artlessly, its element form from the vertices of equilateral triangle with vertices $1,2,3$ and using $\rho_{i}$ for rotations and $\beta_{i}$ for mirror images.

$$
\begin{gathered}
\rho_{0}=\left(\begin{array}{lll}
1 & 2 & 3 \\
1 & 2 & 3
\end{array}\right), \quad \rho_{1}=\left(\begin{array}{lll}
1 & 2 & 3 \\
3 & 1 & 2
\end{array}\right), \\
\rho_{2}=\left(\begin{array}{lll}
1 & 2 & 3 \\
2 & 3 & 1
\end{array}\right) \\
\beta_{1}=\left(\begin{array}{lll}
1 & 2 & 3 \\
1 & 3 & 2
\end{array}\right), \quad \beta_{2}=\left(\begin{array}{lll}
1 & 2 & 3 \\
3 & 2 & 1
\end{array}\right), \\
\beta_{3}=\left(\begin{array}{lll}
1 & 2 & 3 \\
2 & 1 & 3
\end{array}\right)
\end{gathered}
$$

|  | $\rho_{0}$ | $\rho_{1}$ | $\rho_{2}$ | $\beta_{1}$ | $\beta_{2}$ | $\beta_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\rho_{0}$ | $\rho_{0}$ | $\rho_{1}$ | $\rho_{2}$ | $\beta_{1}$ | $\beta_{2}$ | $\beta_{3}$ |
| $\rho_{1}$ | $\rho_{1}$ | $\rho_{2}$ | $\rho_{0}$ | $\beta_{3}$ | $\beta_{1}$ | $\beta_{2}$ |
| $\rho_{2}$ | $\rho_{2}$ | $\rho_{0}$ | $\rho_{1}$ | $\beta_{2}$ | $\beta_{3}$ | $\beta_{1}$ |
| $\beta_{1}$ | $\beta_{1}$ | $\beta_{3}$ | $\beta_{2}$ | $\rho_{0}$ | $\rho_{1}$ | $\rho_{2}$ |
| $\beta_{2}$ | $\beta_{2}$ | $\beta_{1}$ | $\beta_{3}$ | $\rho_{2}$ | $\rho_{0}$ | $\rho_{1}$ |
| $\beta_{3}$ | $\beta_{3}$ | $\beta_{2}$ | $\beta_{1}$ | $\rho_{1}$ | $\rho_{1}$ | $\rho_{0}$ |

Figure 3. Multiplication table of $S_{3}$.
The order of each element of $S_{3}$ we have;

$$
\begin{aligned}
& \left\langle\rho_{0}\right\rangle=\left\{\rho_{0}\right\} \rightarrow\left|\left\langle\rho_{0}\right\rangle\right|=1 \\
& \left\langle\rho_{1}\right\rangle=\left\{\rho_{0}, \rho_{1}, \rho_{2}\right\} \rightarrow\left|\left\langle\rho_{1}\right\rangle\right|=3 \\
& \left\langle\rho_{2}\right\rangle=\left\{\rho_{0}, \rho_{1}, \rho_{2}\right\} \rightarrow\left|\left\langle\rho_{2}\right\rangle\right|=3 \\
& \left\langle\beta_{1}\right\rangle=\left\{\rho_{0}, \beta_{1}\right\} \rightarrow\left|\left\langle\beta_{1}\right\rangle\right|=2 \\
& \left\langle\beta_{2}\right\rangle=\left\{\rho_{0}, \beta_{2}\right\} \rightarrow\left|\left\langle\beta_{2}\right\rangle\right|=2 \\
& \left\langle\beta_{3}\right\rangle=\left\{\rho_{0}, \beta_{3}\right\} \rightarrow\left|\left\langle\beta_{3}\right\rangle\right|=2
\end{aligned}
$$

The order of each element of $S_{3}$ are less than 6. Therefore, $S_{3}$ with operation multiplication is noncyclic group.

Similarly, if the order of Set A is odd greater than 3. If $S_{n}$ be the collection of the permutations of the elements of $A$. Artlessly, its element form from
the vertices of regular polygon with vertices $1,2,3, \mathrm{n}$ and using $\rho_{i}$ for rotations and $\beta_{i}$ for mirror images. The set $S_{n}$ with operation multiplication is non - cyclic group.

If $n$ is even greater than or equal to four. The elements of $S_{n}$ form from the vertices of regular polygon using $\rho_{i}$ for rotations,

## $\beta_{i}$ for mirror images

 and $\mu_{i}$ for diagonal flips. The set $S_{n}$ with operation multiplication are also non - cyclic group.These are the familiar examples of non - cyclic groups. The researcher also investigated some set with operation multiplication which are non - cyclic group. The investigation starts with a residue class modulo $n$ with operation addition. On these sets with operation addition, we could not collect pair of Set and operation which satisfy non - cyclic group. The following are some of the illustrations:

## Illustration1:

Let the given set be $Z_{5}=\{0,1,2,3,4\}$, residue classes modulo 5 with operation addition. The figure below shows the group table of $Z_{5}$ with operation addition.

| + | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 0 |
| 2 | 2 | 3 | 4 | 0 | 1 |
| 3 | 3 | 4 | 0 | 1 | 2 |
| 4 | 4 | 0 | 1 | 2 | 3 |

Figure 4. Group table of $\mathrm{Z}_{5}$ with operation addition.

The order of each element of $Z_{5}$ are:

$$
\begin{aligned}
& \langle 0\rangle=\{0\} \rightarrow|\langle 0\rangle|=1 \\
& \langle 1\rangle=\{0,1,2,3,4\} \rightarrow|\langle 1\rangle|=5 \\
& \langle 2\rangle=\{0,1,2,3,4\} \rightarrow|\langle 2\rangle|=5 \\
& \langle 3\rangle=\{0,1,2,3,4\} \rightarrow|\langle 3\rangle|=5 \\
& \langle 4\rangle=\{0,1,2,3,4\} \rightarrow|\langle 4\rangle|=5
\end{aligned}
$$

Thus, $Z_{5}=\langle 1\rangle=\langle 2\rangle=\langle 3\rangle=\langle 4\rangle$. This means that the elements $1,2,3,4$ generates $Z_{5}$. Therefore, $Z_{5}$ with operation addition is cyclic group.

Illustration 2:
Let consider the $\operatorname{set} Z_{6}=\{0,1,2,3,4,5\}$, residue classes modulo 6 with operation addition. The order of the elements of the given set are:

$$
\langle 0\rangle=\{0\} \rightarrow|\langle 0\rangle|=1
$$

$\langle 1\rangle=\{0,1,2,3,4,5\} \rightarrow|\langle 1\rangle|=6$
$\langle 2\rangle=\{0,2,4\} \rightarrow|\langle 2\rangle|=3$
$\langle 3\rangle=\{0,3\} \rightarrow|\langle 3\rangle|=2$
$\langle 4\rangle=\{0,2,4\} \rightarrow|\langle 4\rangle|=3$
$\langle 5\rangle=\{0,1,2,3,4,5\} \rightarrow|\langle 5\rangle|=6$
Therefore $Z_{6}$ with operation addition is cyclic group generated by 1 , and 5 .

Both illustrations are cyclic groups. If we observe the generator of its group are those elements relatively prime to $n$. If $n$ is greater than or equal to 2 , then $n$ have at least one positive integer relatively prime and less than to it. Therefore, the Set residue classes modulo n with operation addition are cyclic group. Non- zero residue classes modulo n relatively prime to n with operation multiplication

The Set of non- zero residue classes modulo $n$ relatively prime to n is denoted by $Z_{n}^{\prime}$. Let $*$ define as ordinary multiplication and considering $n=$ $\{2,3,4, \ldots, 10\}$.

Table 1. Identification of Cyclic and Non - Cyclic Group Conjectures

| $n$ | Elements | Table |  |  |  |  |  | Order of the elements$\|\langle 1\rangle\|=1$ | Generator <br> /(s)1 | Remarks <br> Cyclic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | \{1\} | $\begin{gathered} x \\ \hline 1 \end{gathered}$ | 1 |  |  |  |  |  |  |  |
| 3 | \{1,2\} | $\begin{aligned} & x \\ & \hline 1 \\ & 2 \end{aligned}$ | $1$ |  |  |  |  | $\begin{aligned} & \|\langle 1\rangle\|=1 \\ & \|\langle 2\rangle\|=2 \end{aligned}$ | 2 | Cyclic |
| 4 | \{1,2\} | $\begin{aligned} & x \\ & \hline 1 \\ & 3 \end{aligned}$ | $1$ |  |  |  |  | $\begin{aligned} & \|\langle 1\rangle\|=1 \\ & \|\langle 3\rangle\|=2 \end{aligned}$ | 3 | Cyclic |
| 5 | $\{1,2,3,4\}$ | $\begin{aligned} & x \\ & \hline 1 \\ & 2 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline 1 \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array}$ | $\begin{aligned} & 2 \\ & \hline 2 \\ & 4 \\ & 1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \hline 3 \\ & \hline 3 \\ & 1 \\ & 4 \\ & 2 \end{aligned}$ | $\begin{aligned} & 4 \\ & \hline 4 \\ & 3 \\ & 2 \\ & 1 \end{aligned}$ |  | $\begin{aligned} & \|\langle 1\rangle\|=1 \\ & \|\langle 2\rangle\|=4 \\ & \|\langle 3\rangle\|=4 \\ & \|\langle 4\rangle\|=2 \end{aligned}$ | 2,3 | Cyclic |
| 6 | $\{1,5\}$ |  <br> 1 <br> 5 | $\begin{aligned} & \hline 1 \\ & \hline 1 \\ & 5 \end{aligned}$ |  |  |  |  | $\begin{aligned} & \|\langle 1\rangle\|=1 \\ & \|\langle 5\rangle\|=2 \end{aligned}$ | 5 | Cyclic |
| 7 | \{1, 2, 3, 4, 5,6\} | $\begin{gathered} \hline x \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{gathered}$ |  |  | $\begin{gathered} \hline 4 \\ \hline 4 \\ 1 \\ 5 \\ 2 \\ 6 \\ 3 \end{gathered}$ | $\begin{aligned} & \hline 5 \\ & \hline 5 \\ & 3 \\ & 1 \\ & 6 \\ & 4 \end{aligned}$ | $\begin{aligned} & \hline 6 \\ & \hline 6 \\ & 5 \\ & 4 \\ & 3 \\ & 2 \\ & 1 \end{aligned}$ | $\begin{aligned} & \|\langle 1\rangle\|=1 \\ & \|\langle 2\rangle\|=3 \\ & \|\langle 3\rangle\|=6 \\ & \|\langle 4\rangle\|=3 \\ & \|\langle 5\rangle\|=6 \\ & \|\langle 6\rangle\|=2 \end{aligned}$ | 3, 5 | Cyclic |
| 8 | \{1, 3, 5,7\} | $x$ <br> 1 <br> 3 <br> 5 <br> 7 | $\begin{array}{\|l\|} \hline 1 \\ \hline 1 \\ 3 \\ 5 \\ 7 \\ \hline \end{array}$ | $\begin{aligned} & \hline 3 \\ & \hline 3 \\ & 1 \\ & 7 \\ & 5 \end{aligned}$ | $\begin{aligned} & \hline 5 \\ & \hline 5 \\ & 7 \\ & 1 \\ & 3 \end{aligned}$ | 7 7 5 3 1 |  | $\begin{aligned} & \|\langle 1\rangle\|=1 \\ & \|\langle 3\rangle\|=2 \\ & \|\langle 5\rangle\|=2 \\ & \|\langle 7\rangle\|=2 \end{aligned}$ | none | Not Cyclic |



In table 1, the following were observed:

1. The largest number of generators is 2 .
2. The multiplication table of modulo 4 and 8 using multiplication operation is the same as the Viergrouppe. The entry on the main diagonal is the identity element.
3. Only $Z_{8}^{\prime}$ is non - cyclic group.

Based on the observation the following conjectures were formed:

1. The largest number of generators for the set of non - zero residue classes modulo $n$ relatively prime to $n$ under operation multiplication is 2 .
2. The non - zero residue classes modulo $n=$ $2^{r}$ where $r \geq 3$ relatively prime to $n$ with operation multiplication are non - cyclic groups.

The illustration below reliably verify conjecture 2 .
Let $n=2^{4}=16$. Then,
$Z_{16}^{\prime}=\{1,3,5,7,9,11,13,15\}$.
Figure 5, shows the multiplication table for $Z_{16}^{\prime}$.

|  | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
| 3 | 3 | 9 | 15 | 5 | 11 | 1 | 7 | 13 |
| 5 | 5 | 15 | 9 | 3 | 13 | 7 | 1 | 11 |
| 7 | 7 | 5 | 3 | 1 | 15 | 13 | 11 | 9 |
| 9 | 9 | 11 | 13 | 15 | 1 | 3 | 5 | 7 |
| 11 | 11 | 1 | 7 | 13 | 3 | 9 | 15 | 5 |
| 13 | 13 | 7 | 1 | 11 | 5 | 15 | 9 | 3 |
| 15 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1 |

$\langle 1\rangle=\{1\} \rightarrow|\langle 1\rangle|=1$
$\langle 3\rangle=\{1,3,9,11\} \rightarrow|\langle 3\rangle|=4$
$\langle 5\rangle=\{1,5,9,13\} \rightarrow|\langle 5\rangle|=4$
$\langle 7\rangle=\{1,7\} \rightarrow|\langle 7\rangle|=2$
$\langle 9\rangle=\{1,9\} \rightarrow|\langle 9\rangle|=2$
$\langle 11\rangle=\{1,3,9,11\} \rightarrow|\langle 11\rangle|=4$
$\langle 13\rangle=\{1,5,9,13\} \rightarrow|\langle 13\rangle|=4$
$\langle 15\rangle=\{1,15\} \rightarrow|\langle 15\rangle|=2$
Figure 5.Multiplication table for $Z_{16}^{\prime}$

The largest order of the elements of $Z_{16}^{\prime}$ is 4. Therefore $Z_{16}^{\prime}$, with operation multiplication is non -cyclic group.

If $n=2^{5}=32$. The largest order of elements of $Z_{32}^{\prime}$ under multiplication is 8 . This implies that $Z_{32}^{\prime}$ is non - cyclic group.

## Direct Product

If $G_{1}, G_{2}, \ldots, G_{n}$ be groups, then the direct product $G_{1} x G_{2} x \ldots, x G_{n}$ denoted by $\prod_{i=1}^{n} G_{i}$ is a group under the operation direct sum and product of the groups $G_{i}$.

Theorem. Let $\left(a_{1}, a_{2}, \cdots, a_{n}\right) \in \prod_{i=1}^{n} G_{i}$. If $a_{i}$ is of finite order $r_{i}$ in $G_{i}$, then the order of $\left(a_{1}, a_{2}, \cdots, a_{n}\right)$ in $\prod_{i=1}^{n} G_{i}$ is equal to the least common multiple of all the $r_{i}$ " [5]. If $r_{i}$ are relatively prime with each other, then $\left|\left\langle\left(a_{1}, a_{2}, \cdots, a_{n}\right)\right\rangle\right|$ is equal to $\left|\prod_{i=1}^{n} G_{i}\right|$. This theorem is true in the direct sum of the groups $G_{i}$.

Table 2. Direct Product of the groups $G_{i}$

|  | Order of the elements | Generator /(s) | Remarks |
| :---: | :---: | :---: | :---: |
| Let $G_{1}=Z_{2}^{\prime}=\{1\}$ and $G_{2}=Z_{2}^{\prime}=\{1\}$, then $G_{1} x G_{2}=\{(1,1)\}$ | $\|\langle(1,1)\rangle\|=1$ | $(1,1)$ | $\begin{aligned} & \hline G_{1}=\text { cyclic } \\ & G_{2}=\text { cyclic } \\ & G_{1} x G_{2}=\text { cyclic } \end{aligned}$ |
| Let $G_{1}=Z_{2}^{\prime}=\{1\}$ and $G_{2}=Z_{3}^{\prime}=\{1,2\}$, then $G_{1} x G_{2}=\{(1,1),(1,2)\}$ | $\begin{gathered} \|\langle(1,1)\rangle\|=1 \\ \|\langle(1,2)\rangle\|=2 \\ (1,2)=(1,2) \\ (1,2)(1,2)=(1,1) \end{gathered}$ | $(1,2)$ | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { cyclic } \\ & G_{1} x G_{2}=\text { cyclic } \end{aligned}$ |
| $\begin{aligned} & \text { Let } G_{1}=Z_{2}^{\prime}=\{1\} \text { and } \\ & G_{2}=Z_{5}^{\prime}=\{1,2,3,4\} \text {, then } \\ & \qquad G_{1} x G_{2}=\{(1,1),(1,2),(1,3),(1,4)\} \end{aligned}$ | $\begin{gathered} \|\langle(1,1)\rangle\|=1 \\ \|\langle(1,2)\rangle\|=4 \\ (1,2)=(1,2) \\ (1,2)(1,2)=(1,4) \\ (1,2)(1,4)=(1,3) \\ (1,2)(1,3)=(1,1) \\ \|\langle(1,3)\rangle\|=4 \\ (1,3)=(1,3) \\ (1,3)(1,3)=(1,4) \\ (1,3)(1,4)=(1,2) \\ (1,3)(1,2)=(1,1) \\ \|\langle(1,4)\rangle\|=2 \\ (1,4)=(1,4) \\ (1,4)(1,4)=(1,1) \end{gathered}$ | $(1,2),(1,3)$ | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { cyclic } \\ & G_{1} x G_{2}=\text { cyclic } \end{aligned}$ |
| Let $G_{1}=Z_{2}^{\prime}=\{1\}$ and $G_{2}=Z_{8}^{\prime}=\{1,3,5,7\}$, then $G_{1} x G_{2}=\{(1,1),(1,3),(1,5),(1,7)\}$ | $\begin{gathered} \|\langle(1,1)\rangle\|=1 \\ \|\langle(1,3)\rangle\|=2 \\ (1,3)=(1,3) \\ (1,3)(1,3)=(1,1) \\ \|\langle(1,5)\rangle\|=2 \\ (1,5)=(1,5) \\ (1,5)(1,5)=(1,1) \\ \|\langle(1,7)\rangle\|=2 \\ (1,7)=(1,7) \\ (1,7)(1,7)=(1,1) \end{gathered}$ | None | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { non-cyclic } \\ & G_{1} x G_{2}=\text { non }- \text { cyclic } \end{aligned}$ |
| Let $G_{1}=Z_{3}^{\prime}=\{1,2\}$ and $\begin{aligned} G_{2}= & Z_{4}^{\prime}=\{1,3\}, \text { then } \\ & G_{1} x G_{2}=\{(1,1),(1,3),(2,1),(2,3)\} \end{aligned}$ | $\begin{gathered} \|\langle(1,1)\rangle\|=1 \\ \|\langle(1,3)\rangle\|=2 \\ (1,3)=(1,3) \\ (1,3)(1,3)=(1,1) \\ \|\langle(2,1)\rangle\|=2 \\ (2,1)=(2,1) \\ (2,1)(2,1)=(1,1) \\ \|\langle(2,3)\rangle\|=2 \\ (2,3)=(2,3) \\ (2,3)(2,3)=(1,1) \\ \hline \end{gathered}$ | none | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { cyclic } \\ & G_{1} x G_{2}=\text { non }- \text { cyclic } \end{aligned}$ |
| Let G_1=Z_3 ${ }^{\wedge^{\prime}}=\{1,2\}$ and G_2=Z_8^'=\{1,3,5,7\}, then $\mathrm{G} \_1 \times \mathrm{xG} 2=\{(1,1),(1,3),(1,5),(1,7),(2,1),(2,3)$, $(2,5),(2,7)\}$ | $\|\langle(1,1)\rangle\|=1$ $\|\langle(1,2)\rangle\|=4$ $(1,2)=(1,2)$ $(1,2)(1,2)=(1,4)$ $(1,2)(1,4)=(1,3)$ $(1,2)(1,3)=(1,1)$ $\|\langle(1,3)\rangle\|=4$ $(1,3)=(1,3)$ $(1,3)(1,3)=(1,4)$ $(1,3)(1,4)=(1,2)$ $(1,3)(1,2)=(1,1)$ $\|\langle(1,4)\rangle\|=2$ $(1,4)=(1,4)$ $(1,4)(1,4)=(1,1)$ $\|\langle(2,1)\rangle\|=2$ $(2,1)=(2,1)$ $(2,1)(2,1)=(1,1)$ $\|\langle(2,2)\rangle\|=4$ $(2,2)=(2,2)$ $(2,2)(2,2)=(1,4)$ $(2,2)(1,4)=(2,3)$ | None | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { cyclic } \\ & G_{1} x G_{2}=\text { non }- \text { cyclic } \end{aligned}$ |


|  | $\begin{gathered} (2,2)(2,3)=(1,1) \\ \|\langle(2,3)\rangle\|=4 \\ (2,3)=(2,3) \\ (2,3)(2,3)=(1,4) \\ (2,3)(1,4)=(2,2) \\ (2,3)(2,2)=(1,1) \\ \|(2,4)\rangle=4 \\ (2,4)=(2,4) \\ (2,4)(2,4)=(1,1) \end{gathered}$ |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Let } G_{1}=Z_{3}^{\prime}=\{1,2\} \text { and } \\ & G_{2}=Z_{6}^{\prime}=\{1,5\}, \text { then } \\ & \quad G_{1} x G_{2}=\{(1,1),(1,5),(2,1),(2,5)\} \end{aligned}$ | $\begin{gathered} \|\langle(1,1)\rangle\|=1 \\ \|(1,5)\rangle \mid=2 \\ (1,5)=(1,5) \\ (1,5)(1,5)=(1,1) \\ \|\langle(2,1)\rangle\|=2 \\ (2,1)=(2,1) \\ (2,1)(2,1)=(1,1) \\ \|\langle(2,5)\rangle\|=2 \\ (2,5)=(2,5) \\ (2,5)(2,5)=(1,1) \end{gathered}$ | None | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { cyclic } \\ & G_{1} x G_{2}=\text { non }- \text { cyclic } \end{aligned}$ |
|  | $\|\langle(1,1)\rangle\|=1$ $\|(1,3)\rangle \mid=2$ $(1,3)=(1,3)$ $(1,3)(1,3)=(1,1)$ $\mid\langle(1,5)\rangle=2$ $(1,5)=(1,5)$ $(1,5)(1,5)=(1,1)$ $\|(1,7)\rangle \mid=2$ $(1,7)=(1,7)$ $(1,7)(1,7)=(1,1)$ $\mid\langle(2,1)\rangle=2$ $(2,1)=(2,1)$ $(2,1)(2,1)=(1,1)$ $\|((2,3)\rangle\|=2$ $(2,3)=(2,3)$ $(2,3)(2,3)=(1,1)$ $\|\langle(2,5)\rangle\|=2$ $(2,5)=(2,5)$ $(2,5)(2,5)=(1,1)$ $\|(2,7)\rangle \mid=2$ $(2,7)=(2,7)$ $(2,7)(2,7)=(1,1)$ | none | $\begin{aligned} & G_{1}=\text { cyclic } \\ & G_{2}=\text { non }- \text { cyclic } \\ & G_{1} x G_{2}=\text { non }- \text { cyclic } \end{aligned}$ |

Based on the table the following conjectures were formed:

1. If the order of $G_{1}$ is one and $G_{2}$ is a cyclic group, then $\prod_{i=1}^{2} G_{i}$ is cyclic group under the operation direct product of the groups $G_{i}$.
2. If the order of $G_{1}$ is one and $G_{2}$ is a non-cyclic group, then $\prod_{i=1}^{2} G_{i}$ is non-cyclic group under the operation direct product of the groups $G_{i}$.
3. If at least two of the $G_{i}$ has an order greater than 1. Then, $\prod_{i=1}^{n} G_{i}$ is non-cyclic group under the operation direct product of the groups $G_{i}$ whenever $G_{i}$ are cyclic or non cyclic group.

Table 3 reliably verify conjecture 3 .
Let $G_{1}=Z_{5}^{\prime}=\{1,2,3,4\}$ and $G_{2}=Z_{8}^{\prime}=$ $\{1,3,5,7\}$, then .
$G_{1} x G_{2}\left\{\begin{array}{c}(1,1),(1,3),(1,5),(1,7),(2,1), \\ (2,3),(2,5),(2,7),(3,1),(3,3), \\ (3,5),(3,7),(4,1),(4,3),(4,5),(4,7)\end{array}\right\}$

Table 3. Largest Order of the Elements of $\prod_{i=1}^{2} G_{i \_}$i under the Operation Direct Product

| $\begin{gathered} \|\langle(1,1)\rangle\|=1 \\ \|\langle(1,3)\rangle\|=2 \\ (1,3)=(1,3) \\ (1,3)(1,3)=(1,1) \\ \|\langle(1,5)\rangle\|=2 \\ (1,5)=(1,5) \\ (1,5)(1,5)=(1,1) \\ \|\langle(1,7)\rangle\|=2 \\ (1,7)=(1,7) \\ (1,7)(1,7)=(1,1) \\ \|\langle(2,1)\rangle\|=4 \\ (2,1)=(2,1) \\ (2,1)(2,1)=(4,1) \\ (2,1)(2,1)(2,1)=(3,1) \\ (2,1)(2,1)(2,1)(2,1)=(1,1) \\ \|\langle(2,3)\rangle\|=4 \\ (2,3)=(2,3) \\ (2,3)(2,3)=(4,1) \\ (2,3)(2,3)(2,3)=(3,3) \\ (2,3)(2,3)(2,3)(2,3)=(1,1) \\ \mid\langle(2,5)\rangle=4 \\ (2,5)=(2,5) \\ (2,5)(2,5)=(4,1) \\ (2,5)(2,5)(2,5)=(3,5) \\ (2,5)(2,5)(2,5)(2,5)=(1,1) \\ \mid\langle(2,7)\rangle=4 \\ (2,7)=(2,7) \\ (2,7)(2,7)=(4,1) \\ (2,7)(2,7)(2,7)=(3,7) \\ (2,7)(2,7)(2,7)(2,7)=(1,1) \end{gathered}$ | $\begin{gathered} \|\langle(3,1)\rangle\|=4 \\ (3,1)=(3,1) \\ (3,1)(3,1)=(4,1) \\ (3,1)(3,1)(3,1)=(2,1) \\ (3,1)(3,1)(3,1)(3,1)=(1,1) \\ \|\langle(3,3)\rangle\|=4 \\ (3,3)=(3,3) \\ (3,3)(3,3)=(4,1) \\ (3,3)(3,3)(3,3)=(2,3) \\ (3,3)(3,3)(3,3)(3,3)=(1,1) \\ \|\langle(3,5)\rangle\|=4 \\ (3,5)=(3,5) \\ (3,5)(3,5)=(4,1) \\ (3,5)(3,5)(3,5)=(2,5) \\ (3,5)(3,5)(3,5)(3,5)=(1,1) \\ \|\langle(3,7)\rangle\|=4 \\ (3,7)=(3,7) \\ (3,7)(3,7)=(4,1) \\ (3,7)(3,7)(3,7)=(2,7) \\ (3,7)(3,7)(3,7)(3,7)=(1,1) \\ \|\langle(4,1)\rangle\|=2 \\ (4,1)=(4,1) \\ (4,1)(4,1)=(1,1) \\ \|\langle(4,3)\rangle\|=2 \\ (4,3)=(4,3) \\ (4,3)(4,3)=(1,1) \\ \mid\langle(4,5)\rangle=2 \\ (4,5)=(4,5) \\ (4,5)(4,5)=(1,1) \\ \|\langle(4,7)\rangle\|=2 \\ (4,7)=(4,7) \\ (4,7)(4,7)=(1,1) \end{gathered}$ |
| :---: | :---: |

The largest order of the elements of $\prod_{i=1}^{2} G_{i}$ under the operation direct product is 4 . This implies that $\prod_{i=1}^{2} G_{i}$ is non - cyclic group.

## IV. CONCLUSION

Through the process of critical investigation on the set $Z_{n}^{\prime}$ with operation multiplication and $\prod_{i=1}^{n} G_{i}$ with the operation direct product of the groups $G_{i}$, the following conjectures were formulated. These derived conjectures were verified true.

The largest number of generators for the set of non - zero residue classes modulo $n$ relatively prime to $n$ under operation multiplication is 2 .

The non - zero residue classes modulo $n=$ $2^{r}$ where $r \geq 3$ relatively prime to $n$ with operation multiplication are non - cyclic groups.

If the order of $G_{1}$ is one and $G_{2}$ is a cyclic group, then $\prod_{i=1}^{2} G_{i}$ is cyclic group under the operation direct product of the group $G_{i}$.

If the order of $G_{1}$ is one and $G_{2}$ is a non-cyclic group, then $\prod_{i=1}^{2} G_{i}$ is non-cyclic group under the operation direct product of the $\operatorname{group} G_{i}$.

If at least two of the $G_{i}$ has an order greater than 1. Then, $\prod_{i=1}^{n} G_{i}$ is non-cyclic group under the operation direct product of the group $G_{i}$ whenever $G_{i}$ are cyclic or non cyclic group.

## V. RECOMMENDATION

All the conjectures formulated reliably verified were correct. Therefore, teachers teaching Abstract Algebra could use those conjectures as an example specifically in cyclic and non-cyclic groups.

## REFERENCES

[1] Renze, J. \& Weisstein, E. W. (2016). "Abstract Algebra."From Math World--A Wolfram Web Resource.http://mathworld.wolfram.com/Abstrac tAlgebra.html.
[2] Jaisingh, L.R. \& Ayres, F. Jr. (2004). Schaum'sOutlines Abstract Algebra 2nd Ed. McGraw-Hill Companies.
[3] Atherton, J. S. (2014). Learning and teaching; Aspectsof cognitive learning theory (On-line: UK). Retrieved July 30, 2014, from http://www.learningandteaching.info/learning/co nstructivism.htm
[4] Moldovan, Ignat, Bălaş-Timar, (2011).Fundamentelepsihologiei II, Mecanisme stimulator energizante, reglatoareşiintegratoare. EdituraUniversităţii"AurelVlaicu", Arad.
[5] Fraleigh, J.B., (2002). A First Course in Abstract Algebra 6th Edition. Addison Wesley Longman Publishing Company Inc.
[6] Zeitz, P. (2007). The Art and Craft of Problem Solving 2nd Edition.John Wiley \& Sons, Inc.

