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**Abstract** – *This study determined the errors committed and difficulties encountered by the students in linear algebra. It also determined the relationship between the two variables. Moreover, problems encountered were also identified. The findings of the study were used as bases in the development of a new instruction plan that will minimize the errors and improve their performance.*

*In determining the errors committed and difficulties encountered, frequency and percentage were used. Likewise, Pearson Product Moment Correlation Coefficient was used to determine the relationship between the two variables. Furthermore, average weighted mean was used to identify the problems encountered.*

*The study revealed that the most common errors of students in linear algebra are obvious computational errors and random response. It was also revealed that students had difficulty in understanding the concept and setting up and manipulating equation employing algebra. Moreover, it was found out that there is a significant relationship between the difficulties encountered and errors committed by the students.*

*The study further revealed that students rarely encountered personal, emotional and problems on professor's instruction in Linear Algebra. In terms of personal problems, students sometimes encountered poor study habits and excessive school activities. In the aspect of emotional problems, they sometimes encountered excessive stress in doing academic tasks and low self-esteem. Moreover, they also sometimes encountered problems on professor's instruction along the non-varied method used and the inability to relate the lesson to real life situation.*

*As a result, a new instruction plan was developed. This instruction plan will minimize the errors committed by students in linear algebra and will eventually improve their performance.*

**Keywords** – *error, error analysis, linear algebra*

## INTRODUCTION

In light of the strong need for upgrading the quality of education in the country, the government has already implemented a transition in the programs of basic education by expanding the ten years cycle to the global standard of twelve years. According to the Department of Education [1], the rationale behind this move of the government is the poor quality of education in the country which is reflected in the low achievement scores of Filipino students especially in mathematics and science. This state of math and science education in the country is evidenced in the Trends in Mathematics and Science Study (TIMSS), (2011) [1] that even with only the science high schools participating in the advanced mathematics category in 2008, the Philippines ranked last. This is supported by Orleans (2007) stating that the current state of science education in the Philippines, particularly in the basic education level lags behind other countries as indicated in the second and third TIMSS that placed the Philippines in disadvantaged positions among participating nations.

The National Achievement Test (NAT) for grade 6 in the school year 2009 – 2010 passing rate is only 69.21%. Although this is already a 24% improvement over the school year 2005-2006 passing rate, further reforms are needed to achieve substantial improvement. The NAT for high school is 46.38% back in 2009-2010, a slight decrease from 47.40% in the school year 2008-2009 [2]

Students also lack the basic competencies for higher education as supported by Ferido and Tan (2009) [4], citing the results of the Scientific, Technological and Environmental Literacy (STEL, n.d.) that there is a decreasing correct answers of students from being familiar with scientific and technological terms to the use of scientific knowledge in problem solving and decision making. Furthermore, this difficulty of students was noted by the Department of Education (2012) citing the skills report of the World Bank of the Philippines that there are serious gaps in critical skills of graduates such as problem solving and creativity.

Due to these alarming situations and issues on the problem solving skills of students, the researchers' attention and interest were caught in order to conduct a study that aims to examine the most common errors encountered by students, particularly in Linear algebra, and find out to what extent they find difficulty on the content areas.

Mathematics is part of our everyday life. Its role is indispensable for it is contributory to all aspects of living. It has been one of the pillars in the academe and is taken by all students in the elementary and high school levels. In tertiary education, it is also a part of the curriculum and serves as a general subject vital in all careers regardless of what program the students are enrolled in. The mathematical skill and reasoning is very fundamental to be learned by any individual. Hence, it is imperative to be numerically competent. Its salient contributions can be seen in any examination, may it be an aptitude test, college

admission test, application for a job, civil service examination, board examination and the like. It is pervasive for the world of work that students must be prepared numerically and reason out quantitatively.

Linear Algebra is one of the branches of mathematics. It mainly deals with linear equations, linear transformations, matrices, determinants. It is central to both pure and applied mathematics. Combined with calculus, linear algebra facilitates the solution of linear systems of differential equations. Techniques from linear algebra are also used in analytic geometry, engineering, physics, natural sciences, computer science, computer animation, and the social sciences particularly economics. Because linear algebra is such a well-developed theory, nonlinear mathematical models are sometimes approximated by linear models. Linear programming has been used to solve problems involving linear algebra and several software products are available for this purpose.

Linear algebra plays an important role in solving many applied problems; it is also a source of challenging theoretical questions in its own right. Techniques from linear algebra are used in areas such as information retrieval, decision making, population dynamics and systems and control theory ([www.hamilton.ie](http://www.hamilton.ie)). With its powerful usage, mathematicians are engaged in a variety of research topics spanning its theory and applications.

Linear algebra is one of the major subjects taken by Bachelor of Secondary Education students majoring in Mathematics (BSE-Math). This is one of the many courses offered by the researchers' institutional affiliations: Pangasinan State University (PSU) and Benguet State University (BSU). As observed, students often commit mistakes in solving. Sometimes, even simple answers are overlooked. As a result, students obtain low grades in the subject. Identifying the most common errors is the key to developing solutions to this problem.

It is hoped that the results of this study will facilitate the ongoing developments of math education in the universities and will aid in providing suitable programs for BSE-Math majors. The results of the study may be found useful for administrators and curriculum planners in enhancing math education, like identifying issues and finding possible solutions to these. This study will be geared towards math educators because it can encourage them to exert more intelligent efforts to plan and implement lessons and activities that will enhance the proficiency of students, particularly in linear algebra. The findings of this study will expectantly serve as basis for math teachers in identifying which math goals and objectives are to be achieved. It can also supply invaluable information for the teaching of math as the results can provide adequate account of the way in which students learn the facts and process of problem solving.

The identification of errors can focus the strength of teaching linear algebra. It can make teachers aware of the need to evaluate their teaching procedures and adjust

instruction according to the diagnosis, thus continually meeting the individual needs and abilities of the students. It can also be a guide to reappraise their present methods and materials in teaching linear algebra.

It is also the goal of this undertaking to be useful to BSE-Math students. Awareness of their errors, difficulties and misconceptions can become an encouragement to exert greater effort in learning the skills needed to hurdle higher math subjects.

### **OBJECTIVES OF THE STUDY**

This study determined the most common errors encountered by students of Bachelor of Secondary Education major in Mathematics in Linear Algebra.

Specifically, the study answered the following questions:

1. What common errors do students commit along the following content areas:
  - a. Linear equations and matrices;
  - b. Determinants;
  - c. Vectors and vector Spaces; and
  - d. Linear Transformations and Matrices?
2. What common difficulties do students encounter on the aforementioned content areas?
3. Is there significant relationship between the errors committed and difficulties encountered by students on the content areas?
4. What are the problems encountered by the students in Math 114: Linear Algebra?
5. What instructional plan may be prepared to minimize errors committed by students in Linear Algebra?

### **MATERIALS AND METHODS**

This study made use of descriptive-correlational research design. However this study is also non-experimental. Descriptive-correlational research design describes an existing relationship between variables and seeks to clarify an understanding of important phenomena by identifying relationships among variables (Fraenkel and Wallen, 2006).

In this study, relationship between the errors committed and the difficulties encountered on the content areas of Linear Algebra was examined. Thus, descriptive-correlational design was deemed most appropriate because it describes the relationships of the two quantitative variables.

### Sources of Data

This study was conducted at PSU BC and BSU. Bachelor of Secondary Education Major in Math Fourth year students who have taken Linear Algebra were the respondents of this study. Students in those year level were the only possible respondents since Linear Algebra is offered every second semester of the third curriculum year in both schools. Sample size was computed using the Slovin's formula at 5 percent margin of error. Forty five respondents were considered for this study.

### Instrumentation and Data Collection

To identify the common errors encountered by the students in the content areas of Linear Algebra, a questionnaire based on the Course Syllabus of Linear Algebra was constructed. The questionnaire has six (6) problem solving questions. The same instrument was used to identify the difficulties encountered by the students.

To identify the problems encountered by the students in Linear Algebra, a survey questionnaire was used. Problems encountered were categorized as emotional problems, personal problems or problems on professor's instruction.

To ensure the content and face validity of the instrument, the questionnaires were submitted to four (4) competent people in the field of Mathematics. The weighted mean of 4.78 indicated that the instrument was highly valid in terms of content validity. Moreover, the instruments were also pre-tested to eleven (11) BS Applied Statistics 4th year students of BSU to ensure reliability.

The researchers sought the permission of the Mathematics Department Chairs of PSU and BSU. Distribution was personally conducted by the researchers. The researchers checked, encoded and summarized the results. The summary of results was used for the statistical treatment.

### Analysis of Data

To determine the errors encountered by the students in the content areas of Linear Algebra, errors were categorized either with wrong operations, with obvious computational errors, has defective algorithm, or random response. Frequency and percentage were then used to identify the common errors to which the students find difficulty.

Moreover, difficulties encountered by students were categorized as either understanding the concept, setting up or manipulating equation employing algebra, solving for variables, formulating the equation representing the problem and determining the answer.

To identify whether there is a significant relationship between the errors committed and difficulties encountered by the students on the subject areas, Pearson Product Moment Correlation Coefficient was used. To determine the problems encountered by the students in Linear Algebra,

weighted mean was used. Weighted means were then interpreted using the following pre-set criteria.

**Table 1**  
**Pre-set Criteria for Problems Encountered by Students in Linear Algebra**

Limits	Descriptive Equivalent
3.21 – 4.00	Always Encountered
2.41 – 3.20	Frequently Encountered
1.61 – 2.40	Sometimes Encountered
0.81 – 1.60	Rarely Encountered
0.00 – 0.80	Never Encountered

Furthermore, the writing of the instruction plan that will minimize the errors of the students was based on the errors committed and problems encountered. Emphasis was given on topics which has the most number of errors. Furthermore, problems identified to be frequently encountered by the students was given much focus. Other previous study results deemed effective to minimize errors was also included.

## RESULTS AND DISCUSSION

### Common Errors Committed by the Students

Table 2 shows the common errors committed along the main topics in Linear algebra namely Linear Equations and Matrices, Determinants, Vectors and Linear Transformation and Matrices

**Table 2: Common Errors Committed by Students in Linear Algebra**

Errors Committed	Topics in Linear Algebra									
	Linear Equations and Matrices			Determinants			Vectors			Linear au
	Count	Percent	Rank	Count	Percent	Rank	Count	Percent	Rank	Count
1. Wrong Operation	5	14.7	2.5	5	14.7	2	0	0	4	3
2. Obvious Computational Error	13	38.24	1	7	20.59	1	1	2.94	3	1
3. Defective Algorithm	5	14.7	2.5	2	5.88	3	2	5.88	2	4
4. Random Response	2	5.88	4	1	2.94	4	9	26.47	1	11
Correct	9			19			6			7
No response	0			0			16			8

#### 1. Wrong Operation

This type of error is ranked 4th under Vectors; ranked 3rd in Linear Transformation and Matrices; 2nd under Determinants; and ranked 2.5 in Linear Equations and Matrices. This can be seen in Table 2. In this particular error, the students were observed to respond by performing an operation other than the one that is required to solve the problem. Most of these are errors where algebraic rules were violated. To address this error, the instructor could give extensive background on methods in manipulating equations.

#### 2. Obvious Computational Error

As reflected on Table 2, the error of Obvious Computational Error is placed 4th in the topic Linear Transformation and Matrices; placed 3rd under the topic Vectors; and placed 1st in both topics Linear Equations and Matrices and Determinants. The students used the appropriate operation, but their responses were based on recalling basic number facts. Carelessness of students is the primary reason for this error. Instructors could advise their students not to be in a hurry to solve the problems which they know because they may commit errors due to excitement. Instructors could encourage the students to review their answers before submitting their papers.

### 3. Defective Algorithm

As gleaned from Table 2, this error is ranked third under Determinants; ranked 2.5 under Linear Equations and Matrices; and ranked second in both Vectors and Linear Transformation and Matrices. The students were observed to employ correct operation and use correct data at the beginning, but made errors in carrying out necessary steps. More procedural examples could be given by the instructor to augment this. Theorems must be stressed for students to be guided.

### 4. Random Response

Random response, as presented in Table 2, is placed 4th under both topics Linear Equations and Matrices and Determinants; and placed 1st under the two remaining topics, Vectors and Linear Transformation and Matrices. This error shows no apparent relationship to the given problem. The students' answers have no bearing on the question. Many students answered "yes" to a question that requires a solution. This shows lack of understanding or inability to recall a certain concept. To address this error, the instructor could incorporate utility of concepts in daily life in order for students to retain the concept.

### Common Difficulties Encountered by the Students

Table 3 exhibits the common difficulties encountered along the main topics in Linear algebra, namely Linear Equations and Matrices, Determinants, Vectors and Linear Transformation and Matrices.

**Table 3. Common Difficulties Encountered by the Students**

Difficulties Encountered	Topics in Linear Algebra									
	Linear Equations and Matrices			Determinants			Vectors			Linear Tr and I
	Count	Percent	Rank	Count	Percent	Rank	Count	Percent	Rank	P
1. Understanding the concept	2	5.88	5	5	14.70	1	31	91.18	1	19
2. Setting up and manipulating equation employing algebra	7	20.59	1	1	2.94	5	2	5.88	2	4
3. Solving for variables	6	17.65	2	3	8.82	3	1	2.94	3	0
4. Formulating the equation representing the problem	5	14.70	3.5	4	11.76	2	0	0	4.5	0
5. Determining the answer	5	14.70	3.5	2	5.88	4	0	0	4.5	4

### 1. Understanding the Concept

This indicates that the students have a hard time in comprehending the basic terms, ideas and procedures in solving problems. This could bring about all the other difficulties that a student encounters. If students do not understand and grasp the concepts of a given topic, they find it difficult to answer items or solve problems needing such concepts. Accordingly, students will find it hard to formulate the equation representing the problem, solve variables, to symbolize phrases. In Table 3, it is summarized that this difficulty is ranked 5th under the topic Linear Equations and Matrices and ranked 1st under all remaining topics of linear algebra under this investigation. This finding agrees with that of the study of [10] stating that the most common difficulty encountered by the students is understanding the concept. More time is needed in discussing certain concepts so this difficulty can be reduced.

### 2. Setting Up and Manipulating Equation Employing Algebra

Difficulty in algebra reveals that mastery in algebra is needed to solve related problems in linear algebra. Students must understand the basic operations and rules in algebra. A more extensive background in the prerequisite of Linear algebra is needed to alleviate this difficulty. In Table 3, this difficulty is shown to be placed 5th under the topic Determinants; placed 2.5 under the topic Linear Transformation and Matrices; placed 2nd under the topic Vectors; and placed 1st Linear Equation and Matrices. The finding is in consonance with that of [9] that the difficulty in algebra is among the top difficulties of students.

### 3. Solving for Variables

The difficulty in solving for variables arises because of the following primary reasons: students cannot define the variables they used in solving, students find it difficult solving for variables because of lack of skill in manipulating simultaneous equations, especially in linear algebra. Since identifying the knowns and unknowns are important in problem solving, instructors could give a clearer definition on the variables. In this study, this difficulty is seen in Table 3 to be ranked 4.5 in Linear Transformation and Matrices; ranked 3rd in both Determinants and Vectors; and ranked 2nd in Linear Equations and Matrices.

### 4. Formulating the Equation Representing the Problem

This shows that the students find it hard to formulate an equation to represent a problem because the student misinterpreted the problem. It is important for the students to have a good grasp of the English language to be able to represent worded statements into math symbols to make an equation. This difficulty is evident in both the topic Linear Transformations and Matrices and Vectors as rank 4.5; among all difficulties, this difficulty is ranked 3.5 under the topic Linear Equations and Matrices; and second under the topic Determinants (Table 3).

### 5. Determining the Answer

The students are observed to be technology dependent. In the case of linear algebra, they rely too much on their calculators that even simple mathematical operations are done on a calculator. Final answers are jeopardized by this. Exercises that will make students independent of technology could be used by instructors. This difficulty is placed 4.5 in Vectors; placed 4th in Determinants; placed 3.5 in Linear Equations and Matrices; and placed 2.5 in Linear Transformation and Matrices (Table 3). This is the same finding as that of [6] that determining the answer is one of the difficulties encountered by students in problem solving.

### **Relationship between the Errors Committed by Students and the Difficulties on the Content Areas**

This study was conducted to determine, among others, the relationship between errors committed and difficulties encountered by students. Table 4 shows the correlation between the errors committed and the difficulties encountered by the students. There are numerous difficulties significantly correlated to the errors.

As gleaned from Table 4, the errors are not significantly related to difficulty 1, understanding the concept. This infers that understanding the concept is not an influence in committing errors. Because the student cannot understand the concept, the student cannot advance in solving the problems concerning the concept.

Evident also from the same table is the significant relation between the errors and the difficulty 2, setting up and manipulating equation employing algebra. Obvious computational error and random response are also significantly correlated to this difficulty. This denotes that algebra is important foundation for solving problems in linear algebra.

The relation of errors to difficulty 3, difficulty in solving for the variables is presented in Table 3. Wrong operation and defective algorithm have significant relations to this difficulty implying that if students have difficulty in solving the variables, they are prone to committing errors like wrong operation and defective algorithm.

**Table 4. Correlation Between the Difficulties Encountered and Errors Committed**

	Difficulty 1		Difficulty 2		Difficulty 3		Difficulty 4		Difficulty 5	
	Understanding the concept		Setting up and manipulating equation employing algebra		Solving for variables		Formulating the equation representing the problem		Determining the answer	
	R	sig	R	Sig	r	sig	r	sig	R	sig
Error 1										
Wrong operation	0.03	0.80	0.21	0.10	0.35**	0.00	-0.22	0.08	0.01	0.93
Error 2										
Obvious computational error	0.06	0.65	0.38**	0.00	0.17	0.17	-0.04	0.78	-0.02	0.90
Error 3										
Defective algorithm	-0.11	0.40	-0.11	0.38	0.39**	0.00	0.37**	0.00	0.22	0.08
Error 4										
Random response	-0.08	0.52	0.33*	0.01	0.02	0.88	-0.11	0.41	-0.05	0.65

The findings reveal that only defective algorithm has a significant relation to difficulty 4, formulating the equation representing the problem. This may be because students cannot associate equations with defining variables.

Finally, the results show that the errors are not significantly related with difficulty 5, determining the answer. This may mean that students commit error even if they have difficulty in determining the answer.

As a summary, there is a significant relationship between difficulties and errors committed by the students. Errors that students commit may be influenced by the difficulties they encounter. The exception is in difficulties in understanding the concept and determining the answer.

### **Problems Encountered by Students in Linear Algebra**

The following tables present the problems encountered by students in Linear Algebra along different categories. Considering the overall means of 1.49, 1.29 and 0.83, the tables reveal that the students had rarely encountered these problems categorized as personal problems, emotional problems and problems on professor's instruction, respectively.

**Table 5 : Personal Problems Encountered by Students in Linear Algebra**

Item	Indicators	WM	DE
1	Lack of interest in the subject.	1.38	RE
2	Negative attitude toward the subject	1.09	RE
3	Poor study habits	1.69	SE
4	Involvement in school activities	1.82	SE
	Overall Mean	1.49	RE
Legend			
3.21 - 4.00 = Always Encountered			
2.41 - 3.20 = Frequently Encountered			
1.61 - 2.40 = Sometimes Encountered			
0.81 - 1.60 = Rarely Encountered			
0.00 - 0.80 = Never Encountered			

Table 5 shows the personal problems encountered by students in linear algebra. It can be gleaned in the table that students seldom encountered problems related to poor study habits and involvement in school activities with weighted means 1.69 and 1.82, respectively.

Students do not see the need to learn about mathematics until they can learn about real life through mathematics. Once students realize the importance of learning mathematics, they will have the initiative to improve their study habits. Developing good study habits will help them succeed in class and achieve their educational goals. Knowing how to study hard and smart could result in the difference between acing a class and just passing. Those who have poor study habits end up flunking in school or getting behind in school.

Meanwhile, students who are over-scheduled in too many activities find that the benefits of participating in out-of-school activities may actually decrease. Overscheduled children may be tired, irritable and show little interest in participation. It is then important for each student who are greatly involved in school activities to have a good time management. This will enable them to balance academics with extra-curricular activities.

**Table 6 : Emotional Problems Encountered by Students in Linear Algebra**

Item	Indicators	WM	DE
1	Excessive stress in doing academic tasks	1.98	SE
2	Low self-esteem or not believing on one's Capabilities	1.64	SE
3	Undesirable development of a relationship between the teacher and student such as attention and affection with each other	0.84	RE
4	Feeling of insecurity	0.93	RE
5	High anxiety in working so hard at winning the teacher's belief on good impression	1.29	RE
6	Feeling of timidity	1.44	RE
7	Frustration over one's inadequacies	1.36	RE
8	Difficulty in establishing desirable relationship with parents and siblings	0.82	RE
Overall Mean		1.29	RE

Table 6 presents the emotional problems encountered by students in linear algebra. It can be gleaned in the table that students feel that they experienced excessive stress in doing academic tasks and that they have low self-esteem as reflected by the weighted means 1.98 and 1.64, respectively.

At school, students feel a range of academic pressure, derived from a need of perfection, worry over grades, parental pressure and tough academic tasks. In dealing with pressure, students need time management, dedication and commitment.

Moreover, when students have low self-esteem, learning is not easy. Therefore, teachers need to work on lowering affective filters to teach the day's lessons by examining current practices that may inhibit students from learning the necessary content. To improve student's self-esteem, teachers need to give students sufficient time to think, process, and rehearse with others before asking a question or requiring an activity to be completed and

employ active lesson activities such as modeling, guided practice, step-by-step explanations, partner work, small-group activities, manipulative activities, and the application games. Every teacher needs to consider nonthreatening and encouraging ways to lower a student's affective filter in order to promote learning. Improving the self-esteem of the student ability to learn the planned content and apply learning toward success in the mathematics curriculum.

**Table 7 : Problems Encountered by Students in Linear Algebra along Professor's Instruction**

Item	Indicators	WM	DE
1	Lacks ability to give clear directions and logical explanations	0.8	NE
2	No effective motivation and introduction	1.13	RE
3	Not creative enough to adapt his/her method to the learner's capability	0.87	RE
4	Lacks the ability to encourage critical and creative though	0.62	NE
5	No visual aids and other examples were used to illustrate the lesson	0.87	RE
6	No sufficient and concrete examples to create meaningful learning experiences	0.64	NE
7	Not asking appropriate and different types of questions that direct students' thinking	0.69	NE
8	One approach, method and strategy in teaching is used	1.58	RE
9	Questions not properly distributed to all learners	0.64	NE
10	Method is not suited to the needs and capabilities of learners	0.76	NE
11	Inability to relate lessons to actual life situations	1.16	RE
12	Not recognizing the value of lifelong learning	0.76	NE
13	No effective use of formative test after teaching	0.56	NE
14	Overlooks to provide students with practical applications of content learned	1.13	RE
15	Not demonstrating in-depth knowledge of the subject matter	0.56	NE
16	Constructed evaluation tools and measures are not valid.	0.47	NE
17	No provision of varied learning tasks	0.82	RE
Overall Mean		0.83	RE

Table 7 reveals problems encountered by students in linear algebra along professor's instruction. It can be gleaned in the table that students rarely encountered problems on linear algebra instructors who used one approach, method and strategy as reflected by its weighted mean of 1.58. However, not all students learn equally when the same strategies are employed and no teaching method is superior particularly in terms of student performance , to another in all learning situations. Moreover, not all students process the new information in the same ways or bring the same skill sets to the learning experience. Some students need extra time to process concepts and look at problems in different ways (Sutton and Krueger, 2002) . Other students need further teaching or teaching presented in multiple ways. Thus, it is imperative for teachers to differentiate instruction to meet those needs within the classroom.

It can also be gleaned in the table that students rarely encountered problems on instructor's inability to relate lessons to actual life situations as reflected by its weighted mean of 1.16. If teachers are stressing simply the

“skills and drills” of mathematical concepts without helping students apply these procedures and formulas to real-world situations, they are doing students a great disservice. When students study a mathematical concept as embodied by a real event or place, they understand and remember it. Application of abstract concepts to real life is a powerful instructional tool.

Teachers of mathematics must put concepts into real-life context for students in order for them to understand the concepts and make them personal. “When mathematics evolves naturally from problem situations that have meaning to children and are regularly related to their environment, it becomes relevant and helps children link their knowledge to many kinds of situations” (NCTM, 1989)<sup>[15]</sup>. This real-life context, which is necessary for developing student understanding, comes from integrating other subjects into mathematics instruction.

It is also worthy to note that linear algebra instructors use valid evaluation tools and demonstrate in-depth knowledge on the subject matter as reflected by weighted means of 0.47 and 0.56, respectively.

How we assess students—both formatively and summatively, have enormous implications for what we teach and how effectively we teach it. Assessment of learning needs to be carefully constructed so that the information upon which decisions are made is of the highest quality. Thus it is a must for every teacher to use valid evaluation tool.

Teachers need to understand subject matter deeply and flexibly so they can help students map their own ideas, relate one idea to another, and re-direct their thinking to create powerful learning. Teachers also need to see how ideas connect across fields and to everyday life. These are the building blocks of pedagogical content knowledge.

### **CONCLUSION AND RECOMMENDATION**

Based on the findings of the study, the following conclusions were drawn:

1. The most common error committed by students among all topics under Linear Algebra is the obvious computational error followed by random response. The students were observed to use the appropriate operation, but their responses were based on recalling basic number facts. Students were careless in answering the questions maybe due to excitement or haste. Many of the students' answers have no bearing on the question. Lack of understanding or inability to recall a certain concept is evident with these.
2. The most common difficulty encountered by students among all topics under Linear Algebra is understanding the concept followed by setting up and manipulating equation employing algebra. Students were observed to have a hard time in realizing the basics terms, ideas

and procedures in solving problems. This leads to other difficulties in formulating the equation representing the problem, solving variables. Difficulty in algebra reveals that mastery in algebra is needed to solve related problems in linear algebra. Students must understand the basic operations and rules in algebra.

3. The significant relationship between the difficulties encountered and errors committed by the students implies that the errors committed by the students are affected by the difficulties they encounter. However, results do not show this for determining the answer and understanding the concept.
4. Students rarely encountered personal, emotional and problems on professor's instruction in Linear Algebra.
5. A new learning plan is developed. This learning plan will minimize the errors committed by students in linear algebra and will eventually improve their performance, thus, deemed necessary.

### **Recommendations**

Based on the conclusions of the study, the following recommendations are offered:

1. Instructors could advise their students not to be in a hurry to solve the problems which they already know by heart to avoid errors caused by haste. Instructors could encourage the students to review their answers before submitting their papers. Instructors could incorporate applications of concepts in daily life in order for students to retain the concept better.
2. Instructors could give a more extensive background in the prerequisite of Linear algebra to alleviate difficulty. More time is needed in discussing certain concepts.
3. In as much as the difficulties encountered are significantly correlated to errors committed by the students, instructors of the prerequisite subjects for Linear Algebra could give a more in-depth approach in discussing the basics needed to understand how to solve problems in Linear Algebra.
4. Students rarely encountered personal, emotional and problems on professor's instruction in Linear Algebra. Thus, there is a need to minimize these problems to improve their performance.
5. The newly developed instruction plan will minimize the errors committed by students in linear algebra, thus, it is deemed appropriate to use.

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